LETTERS TO THE EDITOR

To the editor:

Transition to Annular Flow in Vertical Upward Gas-Liquid Flow

In a recent publication, Taitel, Barnea & Dukler [*AIChE J.*, **26**, 345 (1980)] reasoned that for upward gas-liquid flow, annular flow cannot exist unless the gas velocity in the gas core is sufficiently high to lift the entrained droplets. Otherwise, these droplets will fall back and accumulate to form a bridge giving rise to the churn or slug flow patterns. In carrying out the analysis, they applied the method of Hinze (1955) for the maximum stable drop size and considered the situation when drops of this maximum stable size is suspended. For the minimum gas velocity U_G required to suspend a droplet, they used

$$U_G = \frac{2}{\sqrt{3}}\left[\frac{g(\rho_L - \rho_G)d}{\rho_G C_d}\right]^{1/2} \qquad [1]$$

where g is the gravitational acceleration, ρ_L and ρ_G the liquid and gas density respectively, d the droplet diameter, and the drag coefficient C_d was chosen as being equal to 0.44. In using [1] with $C_d = 0.44$, fluid circulation within the droplet and the distortion in shape had not been accounted for and the value of $C_d = 0.44$ is completely outside the range of C_d for droplets obtained experimentally by Van Der Leeden et al (1955). Using the value $U_G = 15\ \mathrm{ms^{-1}}$ derived by Taitel et al as being the point of annular transition, [1] gives d = 10 mm for the air-water system. The consideration of the velocity of the maximum stable drop size and the use of [1] for the annular transition is inappropriate.

As a liquid droplet falls through a second fluid, its shape changes with its size (Brodkey 1967). As the droplet size is increased, it will progressively flatten on the bottom while the upper part remains a hemispherical shape. In the case of water, the droplet becomes flatter and flatter as the droplet diameter increases to greater than about 5 mm when the droplet velocity tends to an asymptotic terminal velocity. This velocity is termed the large drop terminal velocity U_t (Porter & Wong 1969).

For air-water upward flow, consider the presence of a droplet of diameter greater than 5 mm. This droplet may be suspended if the gas core velocity is equal to the large drop terminal velocity. Assume that the gas core velocity is in excess of the large drop terminal velocity, then, according to the reasonings of Taitel et al, annular flow will take place. This droplet may be stable or unstable as may be determined using the criteria of Hinze. However, the stability of the droplet really is of little relevance to the consideration of the transition to annular flow. If the droplet is stable, it will continue to ascend and annular flow will be maintained. If the droplet is unstable, it shatters to form a number of smaller droplets. The terminal velocity of any of these newly formed droplets will be less than or equal to the large drop terminal velocity, U_t, and hence, all these newly formed droplets will also continue to rise and annular flow will still be maintained.

Based on the above reasonings, the large drop terminal velocity is the parameter to be considered. In drawing the analogy between the bubbling-to-spray transition on sieve trays and the annular transition in upward gas-liquid flow, Chen et al (1982) have correctly used the large drop terminal velocity. Porter & Wong (1969) gave the large drop terminal velocity as

$$U_t = 0.317(\rho_L/\rho_G)^{0.5}\ (\mathrm{ms^{-1}}) \qquad [2]$$

which has shown to be valid for the liquids iso-octane, hexadecane and water (liquid density ρ_L ranging between 700 and 1000 kg m^{-3}) falling through the atmosphere, water droplets of up to 10 mm diameter in size, and 6 mm diameter water droplets falling in air at various pressures such that the gas density ρ_G varied between 0.6 and 1.2 kg m^{-3}.

It is interesting to point out that if [2] is expressed as

$$U_t = 0.317\,[\,|\rho_D - \rho_C|\,\rho_C]^{0.5}\ (\mathrm{ms^{-1}}) \qquad [3]$$

where the subscripts D and C refer to the discontinuous and continuous phase respectively and the absolute value $|\rho_D - \rho_C|$ is used in the evaluation of [3]. A comparison of [3] with the data of Ramshaw & Thornton (1967) and Stewart & Thornton (1967) indicates that [3] is valid also for liquid droplets rising or falling in an immiscible liquid media with a positive $(\rho_D - \rho_C)$ indicating motion in the direction of the gravitational acceleration and vice versa.

Hence, the annular flow transition will take place if the gas velocity U_G is greater than the large drop terminal velocity as given by [2]. For the air-water system at atmospheric conditions, this occurs at $U_G > 9\ \mathrm{ms^{-1}}$, as against $15\ \mathrm{ms^{-1}}$, the corresponding value for annular flow transition given by Taitel et al. Brodkey (1967) also gave a terminal velocity for water drops similar to that predicted by [2]. Wallis (1974) presented drop terminal velocities in terms of Ku, the Kutateladse number. For water drops falling in air, this occurred at Ku = 1.97, which gives a U_G value in agreement with that predicted using [2]. The analysis of Taitel et al gave, for the annular transition corresponding to the maximum stable drop size, a Ku value of 3.1. The Kutateladse number does not appear to be a general representation of drop terminal velocities.

The use of $U_G > 9\ \mathrm{ms^{-1}}$ for the annular flow transition in the air-water system in fact agrees better with the data shown in Taitel et al. The flow pattern map of Hewitt and Roberts (1969) gave, for the air-water annular flow transition, a constant value of $\rho_G U_G^2 = 100\ \mathrm{kg\ m^{-1}\ s^{-2}}$ except at the very low liquid range. This value of $\rho_G U_G^2$ gives, for air at atmospheric conditions, an U_G which is in exact agreement with that predicted using [2]. The flow pattern map of Spedding & Nguyen (1980) also compare well with the modification given here. In addition, considering the steam-water system and assuming that [2] is still valid, the prediction is also in agreement with the steam-water annular transition at 500 and 1000 psia given in Hewitt & Roberts (1969).

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Reply:

In response to Chen and Speddings communication, "Transition to Annular Flow in Vertical Upward Gas-Liquid Flow," each of the models suggested in our paper "Modelling Flow Pattern Transitions for Steady Upward Gas-Liquid Flow in Vertical Tubes" by Taitel, Bornea and Dukler [*AIChE J*, **26**, 345 (1980)] are subject to refinement and improvement, and suggestions along these lines are welcome. In this case, Chen and Spedding agree with the basic mechanism we propose for this transition, namely, that the gas velocity must be large enough to lift the largest drop. They differ in how to calculate this velocity. In the end, a careful comparison with data provides proof of the pudding!

Chen and Spedding state that their model (Equation 2) gives better agreement with our data than does our model. In fact, this is not quite true. Their Equation 2 gives somewhat better agreement with our data taken in the 5.1 cm diameter pipe, while our model does better with the data from the 2.5 cm diameter pipe.

As to the data from the Hewitt-Roberts report (1969), the comparisons do not seem as clear cut as indicated. If one compares predicted transition velocities with *the data for* air-water (rather than using the empirical correlation), this is the result:

DATA:	>6.2 m/s
Chen and Spedding:	5.0 m/s
Taitel et al:	7.9 m/s.

The data are insufficient to locate the transition curve, and the transition velocity must be *greater* than 6.2 m/sec. A similar comparison with the steam-water data there is as follows:

Data:	2.0 m/s
Chen and Spedding:	1.4 m/s
Taitel et al:	2.2 m/s.

The evidence doesn't seem to support the proposed change in the model, especially if one recognizes that this transition boundary is not a sharp one easy to discern visually.

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ERRATA

In "Effects of London-van der Waals Forces on the Thinning of a Dimpled Liquid Film as a Small Drop or Bubble Approaches a Horizontal Solid Plane," by J. D. Chen and John C. Slattery, [*AIChE J* **28**, 955 (1983)] the first result on page 959 should read:

a) The initial profiles of Figures 2 through 6 are identical, indicating the absence of the effect of the disjoining pressure.

The second sentence under this result should be deleted.

The first sentence of the caption for Figure 11 should read: Comparisons of the predictions of the present theory (R_h = 1.96, B = −0.005, m = 4, C = 5.05, —●—), ...

In "Fine Structure of the CSTR Parameter Space" by V. K. Kwong and T. T. Tsotsis [*AIChE J.*, 29, 343 (1983)] the following should appear after the reference to Figure 3 on page 344:

(The types of dynamic behavior shown in Figure 3 are formally correct only if one neglects a normally narrow region of $D\alpha$ numbers around the saddle-point separatrix loop type bifurcation point on the upper steady-state branch, which is often characterized by dynamic behavior—Kwong (1982). Theoretically, however, the saddle-point separatrix loop type bifurcation behavior shown in Fig. 3, as well as in regions IV_C and IV_D of Williams and Calo (1981) and in region IV_A of Uppal, Ray and Poore (1974) is topologically incorrect since the saddle-point separatrix loop is unstable and a stable limit cycle cannot annihilate into an unstable saddle-point separatrix loop. A detailed account of the complex issues and types of bifurcation associated with saddle and saddle-node separatrix loops exceeds the scope of this publication and will be presented in a forthcoming publication.)

The following literature citation should appear on page 347:

Kwong, V. K., "Bifurcation Phenomena in Lumped Parametric Arrhenius Type Reaction Systems with Finite Activation Energies," MS Thesis, University of Southern California, December 1982.